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# Discrete Optimization for Vibration Design of Composite Plates by Using Lamination Parameters

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# Discrete Optimization for Vibration Design of Composite Plates by Using Lamination Parameters

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**Abstract**

A design method is proposed to optimize the stacking sequence of laminated composite plates for desired vibration characteristics. The objective functions are the natural frequencies of the laminated plates, and three types of optimization problems are studied where the fundamental frequency and the difference of two adjacent frequencies are maximized, and the difference between the target and actual frequencies is minimized. The design variables are a set of discrete values of fiber orientation angles with prescribed increment in the layers of the plates. The four lamination parameters are used to describe the bending property of a symmetrically laminated plate, and are optimized by a gradient method in the first stage. A new technique is introduced in the second stage to convert from the optimum four lamination parameters into the stacking sequence that is composed of the optimum fiber orientation angles of all the layers. Plates are divided into sub-domains composed of the small number of layers and designed sequentially from outer domains. For each domain, the optimum angles are determined by minimizing the errors between the optimum lamination parameters obtained in the first step and the parameters for all possible discrete stacking sequence designs. It is shown in numerical examples that this design method can provide with accurate optimum solutions for the stacking sequence of vibrating composite plates with various boundary conditions.



**Keywords**

Composite, laminated plate, lamination parameter, optimization, vibration, natural frequency, layerwise optimization

## 1. Introduction

Advantages of advanced composite materials in industrial use are getting more obvious in the recent global technical trends. Among various composites, laminated fibrous polymer composites are finding a wide range of applications in structural design, especially for light-weight structures that have tight stiffness and strength requirements. They are attractive replacements for conventional metal

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plates in aerospace and automobile industries where high strength/weight and stiffness/weight ratios are desired. In contrast, disadvantages occur in their complex design process and accompanying analytical process considering the material anisotropy and stacking flexibility. It may also be mentioned that light-weight structures are typically exposed to severe vibration environments and the design for optimizing anti-resonance performance (e.g., by maximizing the natural frequency) becomes more important than before in composite structural design. The literature survey shows that the free vibration of flat plates has been extensively studied in the past.

The works of Bert [1, 2] are the earliest on the optimization of stacking sequence. The optimal sequences were determined, but the method had the drawback of requiring *a priori* knowledge of the fundamental frequency in terms of the stiffnesses of the plate. Kam and Chang [3] determined the optimal stacking sequences for the plates with maximum damping by maximizing the specific damping capacity of the plates *via* a multi-start global optimization technique. Adali and Verijenko [4] determined the minimum cost design of hybrid laminated plate subject to a constraint on the fundamental frequencies or frequency separation using discrete sets of available ply angles as a linear optimization problem. Genetic Algorithms (GAs) which mimic the evolutionary principles and chromosomal processing in natural genetics are used extensively for the stacking sequence optimization problem of laminated plate. Riche and Haftka [5] optimized stacking sequence of laminated plate to maximize buckling load using an integer coded GA, where the buckling problem is a very similar problem to the frequency problem. As for the recent reports optimizing the stacking sequence by GA, Pelletier and Vel [6] implemented multi-objective optimization to maximize a load carrying capacity and minimize mass of laminated plate, and to maximize the axial and hoop rigidities and minimize the mass of a cylindrical pressure vessel. On the other hand, Narita and Turvey [7–9] proposed a layerwise optimization (LO) procedure which has a simple optimization algorithm focusing on basic physical observation, where optimum stacking sequences are determined to maximize fundamental frequency and buckling load by the layerwise procedure effectively.

Lamination parameters which are explained in the literature [10] are widely employed in optimization problems of laminated plate as design variables. They are intermediate parameters describing the plate properties in the simple form. When the bending problem without in-plane coupling is considered, the bending stiffnesses for the whole thickness of plate are defined by only four parameters. This means that the lamination parameters are independent of the number of laminates and, therefore, they are commonly accepted as useful parameters to study the properties of laminated plate. Miki and his co-workers proposed a graphical method (which is described in Ref. [10]) using a feasible region of two lamination parameters. Fukunaga *et al.* [11–13] optimized lamination parameters by a gradient method and then derive optimum stacking sequence from corresponding lamination parameters by exploiting geometrical features of feasible regions. Grenestedt [14] and Serge [15] also determined optimum stacking sequence using graphical method. Todoroki and his co-workers [16–18] combined GA with fractal branch-and-bound method or response surface approximation, and optimized effectively laminated plates by assigning lamination parameters as design variables. Kameyama and Fukunaga [19] also used GA with lamination parameters and designed composite plate wings. Furthermore, due to recent development of manufacturing techniques of the laminated composites, Setoodeh and his co-workers [20–22] determined the optimum lamination parameter distributions in the laminated plate to optimize plate compliances, frequency properties, buckling load. These plates with variable stiffness at different locations in the plates are called variable-stiffness plates [23].

All the aforementioned literature references dealt with limited cases of angle-ply laminates or rough increment angles for limited boundary conditions, and did not focus on converting from the lamination parameters into actual stacking sequence. Especially in the literature [20–23], it was confirmed that the variable-stiffness plate indicated improved properties, but optimum fiber shapes were not determined for laminated plates. Generally, with use of the lamination parameters, the design problem is divided into two parts: the first is a relatively easy part to optimize the lamination parameters for the desired plate property, and the second part is to convert from the optimum lamination parameters into the stacking sequence that is composed of the optimum fiber orientation angles of all the layers.

The primary objective of this study is to propose an optimization method to obtain the optimum stacking sequence for vibration of laminated composite plates. The analysis is based on the classical lamination theory for thin plates, and the four lamination parameters are used to describe the bending property of the laminated plates. In the first part, following the established technique, the optimum solutions for the lamination parameters are determined by a gradient method in mathematical programming. The objective functions considered are the fundamental natural frequency and the difference of two adjacent frequencies where both functions are maximized. The difference of the calculated frequency and the target frequency is also minimized to find the stacking sequence of the plates with desired frequency within the physically possible range.

In the second part of the proposed method, a new technique is proposed to convert from the optimum four lamination parameters into the stacking sequence. One has not been able to solve for the $N$-number of optimum fiber orientation angles in all $N$ layers from the optimized four lamination parameters. This is a mathematical problem where one-to-one mapping does not exist. For this, Yamazaki [24] and Autio [25] applied GA methods to determine corresponding lay-ups to the optimum lamination parameters. In those methods, the GA does not require the implementation of the structural analysis and successfully reduces calculation time when it is compared with the conventional GA, which directly assigns fiber orientation angles to design variables [5].

In this study, an angle deriving technique from lamination parameters with more straightforward processes is proposed. The present method obtains the corresponding lay-up by minimizing the errors between the optimum parameters and the parameters for all possible discrete stacking sequence designs. For many layered plates, the LO concept [7] is employed to avoid high calculation efforts and to reduce dimensions of the problem from the whole search domain into sub-domains. This makes the present method less sensitive to increase of the number of layers than the method given in references [24, 25] since the present method solves the many layer problem by iterating the sub-domain problems with a small number of design variables. It is shown in numerical examples that this new design method can provide accurate optimum solutions for the stacking sequence of vibrating composite plates with various boundary conditions.



## 2. Vibration Analysis of Laminated Plates

### 2.1. Laminated Plate and Lamination Parameters

Figure 1 shows a laminated rectangular plate in the co-ordinate systems O-$xyz$ and in each layer the major and minor principal material axes are denoted by the $L$ and $T$ axes. The dimension of a whole plate is given by $a \times b \times h$ (thickness). The plate considered is limited to symmetric laminates, and the total number of layers is defined as $2K$ (i.e., $K$ layers in the upper (lower) half cross-section). Free vibration of such symmetrically laminated thin plates is governed in the classical plate theory by

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2\,\partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3\,\partial y} + 4D_{26}\frac{\partial^4 w}{\partial x\,\partial y^3} + \rho\frac{\partial^2 w}{\partial t^2} = 0, \tag{1}$$

where $w$ is the deflection and $\rho$ is a mean mass per unit area of the plate. The $D_{ij}$ ($i$, $j = 1$, 2 and 6) are the bending stiffnesses of the symmetric laminate defined by

$$\begin{Bmatrix} D_{11} \\ D_{22} \\ D_{12} \\ D_{66} \\ D_{16} \\ D_{26} \end{Bmatrix} = \begin{bmatrix} U_1 & W_1 & W_2 \\ U_1 & -W_1 & W_2 \\ U_4 & 0 & -W_2 \\ U_5 & 0 & -W_2 \\ 0 & W_3/2 & W_4 \\ 0 & W_3/2 & -W_4 \end{bmatrix} \begin{Bmatrix} 1 \\ U_2 \\ U_3 \end{Bmatrix}, \tag{2}$$

where $W_i$ ($i = 1$, 2, 3 and 4) are the lamination parameters [10] and the stiffness invariants $U_i$ ($i = 1$, 2, 3, 4 and 5) are as defined by

$$\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} = \begin{bmatrix} 3/8 & 3/8 & 1/4 & 1/2 \\ 1/2 & -1/2 & 0 & 0 \\ 1/8 & 1/8 & -1/4 & -1/2 \\ 1/8 & 1/8 & 3/4 & -1/2 \\ 1/8 & 1/8 & -1/4 & 1/2 \end{bmatrix} \begin{Bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \end{Bmatrix}. \tag{3}$$

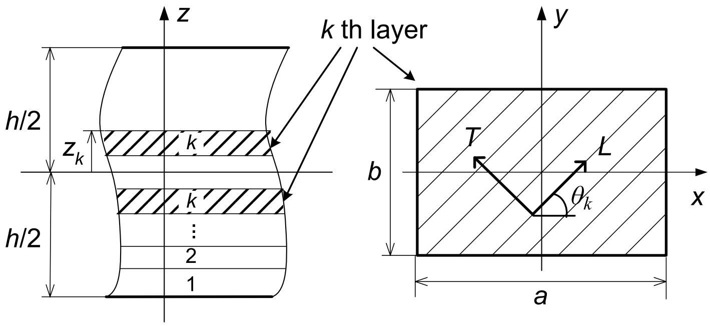


**Figure 1.** Symmetrically laminated rectangular plate and co-ordinate system.



The $Q_{ij}$ ($i, j = 1, 2$ and 6) are elastic constants

$$Q_{11} = \frac{E_L}{1 - \nu_{LT}\nu_{TL}}, \qquad Q_{22} = \frac{E_T}{1 - \nu_{LT}\nu_{TL}},$$
$$Q_{12} = \nu_{TL} Q_{11} = \nu_{LT} Q_{22}, \qquad Q_{66} = G_{LT}, \tag{4}$$

where $E_L$ and $E_T$ are moduli of longitudinal elasticity in the $L$ and $T$ directions, respectively, $G_{LT}$ is a shear modulus and $\nu_{LT}$, $\nu_{TL}$ are the Poisson ratios.

Natural frequency is normalized as a frequency parameter

$$\Omega = \omega a^2 \left( \frac{\rho}{D_0} \right)^{1/2}, \tag{5}$$

where $\omega$ is a radian frequency of free vibration and $D_0 = E_T h^3/12(1 - \nu_{LT}\nu_{TL})$ is a reference stiffness. Lamination parameters are written as follows for a symmetric laminate with respect to the mid-plane.

$$W_1 = \frac{24}{h^3} \int_0^{h/2} \cos 2\theta z^2 \, \mathrm{d}z, \qquad W_2 = \frac{24}{h^3} \int_0^{h/2} \cos 4\theta z^2 \, \mathrm{d}z,$$
$$W_3 = \frac{24}{h^3} \int_0^{h/2} \sin 2\theta z^2 \, \mathrm{d}z, \qquad W_4 = \frac{24}{h^3} \int_0^{h/2} \sin 4\theta z^2 \, \mathrm{d}z. \tag{6}$$

When $W_i$ ($i = 1, 2, 3$ and 4) are normalized by

$$\xi_k = \frac{z_k}{(h/2)}, \tag{7}$$

then the normalized lamination parameters are written as

$$\begin{Bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{Bmatrix} = \sum_{k=1}^{K} (\xi_k^3 - \xi_{k-1}^3) \begin{Bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \\ \sin 2\theta_k \\ \sin 4\theta_k \end{Bmatrix}. \tag{8}$$

One should note that the lamination parameters are dependent on each other. The relationships among the four lamination parameters are expressed as

$$\begin{aligned} &W_1^2 + W_3^2 \leqslant 1, \\ &(W_2 - W_1^2 + W_3^2)^2 + (W_4 - 2W_1W_3)^2 \leqslant (1 - W_1^2 - W_3^2)^2. \end{aligned} \tag{9}$$

*2.2. Free Vibration Analysis of Laminated Plates*

For the small amplitude (linear) free vibration of a thin plate, the deflection $w$ may be written as

$$w(x, y, t) = \bar{w}(x, y) \sin \omega t, \tag{10}$$

where $\bar{w}$ is an amplitude. The maximum strain energy due to bending is expressed by

$$U_{\max} = \frac{1}{2} \iint_A \{\kappa\}^{\mathrm{T}} [D_{ij}] \{\kappa\} \, \mathrm{d}A, \tag{11}$$

where $\{\kappa\}$ is a curvature vector

$$\{\kappa\} = \left\{ -\frac{\partial^2 \bar{w}}{\partial x^2} \quad -\frac{\partial^2 \bar{w}}{\partial y^2} \quad -2\frac{\partial^2 \bar{w}}{\partial x \, \partial y} \right\}^{\mathrm{T}}. \tag{12}$$

The maximum kinetic energy is given by

$$T_{\max} = \frac{1}{2} \rho \omega^2 \iint_A \bar{w}^2 \, \mathrm{d}A. \tag{13}$$

In the Ritz method, the amplitude is assumed in the form

$$\bar{w}(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{mn} X_m(x) Y_n(y), \tag{14}$$

where $A_{mn}$ are unknown coefficients, and $X_m(x)$ and $Y_n(y)$ are the functions modified so that any kinematical boundary conditions are satisfied at the edges with ‘boundary indices’ [26].

After substituting equation (14) into the sum of energies (11) and (13), the stationary value is obtained by

$$\frac{\partial (T_{\max} - U_{\max})}{\partial A_{mn}} = 0 \quad (m = 0, 1, 2, \ldots; n = 0, 1, 2, \ldots). \tag{15}$$

The minimizing process gives a set of linear simultaneous equations in terms of the coefficients $A_{mn}$, and the eigenvalues $\Omega$ may be extracted by existing computer subroutines. This analytical procedure is a standard routine of the Ritz method but the special form of polynomials can satisfy arbitrary sets of kinematical boundary conditions [26].

## 3. Design of Laminated Plates

### 3.1. First Stage: Optimization of the Lamination Parameters

A symmetrically laminated plate is considered for the optimum stacking sequence, and four lamination parameters are used as design variables under the constraints (9). As an optimizer, the modified feasible direction method is adopted, with the golden section method in one-dimensional search, in the ADS program [12, 27].

Three types of optimization problems can be formulated as follows:

(1) Maximizing the fundamental frequency

$$\begin{aligned}&\text{Maximize } \Omega_1(W_1, W_2, W_3, W_4)\\&\text{Subject to } W_1^2 + W_3^2 \leqslant 1 \text{ and}\\&\qquad (W_2 - W_1^2 + W_3^2)^2 + (W_4 - 2W_1W_3)^2 \leqslant (1 - W_1^2 - W_3^2)^2.\end{aligned} \tag{16}$$

(2) Maximizing the difference between the first two frequencies

$$\begin{aligned}&\text{Maximize } \Omega_2(W_1, W_2, W_3, W_4) - \Omega_1(W_1, W_2, W_3, W_4)\\&\text{Subject to } W_1^2 + W_3^2 \leqslant 1 \text{ and}\\&\qquad (W_2 - W_1^2 + W_3^2)^2 + (W_4 - 2W_1W_3)^2 \leqslant (1 - W_1^2 - W_3^2)^2.\end{aligned} \tag{17}$$

(3) Minimizing the difference between the target and actual frequencies

$$\begin{aligned}&\text{Minimize } \{\Omega_{\text{target}} - \Omega_1(W_1, W_2, W_3, W_4)\}^2\\&\text{Subject to } W_1^2 + W_3^2 \leqslant 1 \text{ and}\\&\qquad (W_2 - W_1^2 + W_3^2)^2 + (W_4 - 2W_1W_3)^2 \leqslant (1 - W_1^2 - W_3^2)^2.\end{aligned} \tag{18}$$

Types (1) and (2) are intended in common engineering situations to avoid resonance of the plate from external frequencies by maximizing the fundamental frequency and the difference between the first and second frequencies, respectively. Type (3) has more positive technical meanings to design the natural frequency within the physically possible frequency range.

### 3.2. Second Stage: Determining Discrete Fiber Orientation Angles in the Layers

In the discrete optimization problem where each layer has 36 discrete orientation angles with an increment 5°, the combination of all sets of orientation angles is $36^4 = 1\,679\,616$ for a symmetrically laminated 8-layered plate ($K = 4$). It is of course impractical to repeat the frequency analysis so many times.

As discussed previously, it is relatively easy to determine the optimal values of lamination parameters in the first stage for a given condition. However, one faces serious difficulty in the second stage by actually transforming from the obtained values of lamination parameter to the fiber orientation angles.

The present work proposes a technique to derive a set of fiber orientation angles from the lamination parameters. In the technique, all combinations of lamination parameters are calculated from all possible combinations of discrete fiber orientation angles of laminated plates. This is an easy task to calculate all possible values of lamination parameters for all the combinations of the angles, because they are expressed only in terms of sine and cosine functions. Then optimum stacking sequences are determined by comparing the optimum lamination parameters (obtained by the mathematical programming) with the lamination parameters for all possible combinations, and minimizing the difference expressed by

$$E = (W_1 - W_1^*)^2 + (W_2 - W_2^*)^2 + (W_3 - W_3^*)^2 + (W_4 - W_4^*)^2, \qquad (19)$$

where $W_i$ ($i = 1, 2, 3, 4$) are the optimum lamination parameters determined in the first stage and $W_i^*$ ($i = 1, 2, 3, 4$) are all possible discrete parameter values. When optimum stacking sequences are once determined, a vast number of $E$ values are sorted in ascending order. It is important to use an effective sorting technique because a vast number of combinations exist which directly affect the calculation cost. In the present study, one quick sorting technique [28] reduced the time in the sorting calculation.

Since the calculation of $W_i$ by the mathematical programming may involve numerical errors, the feedback of sorted orientation angles with respect to $E$ values is introduced to guarantee the accuracy of the design result. Therefore, a certain number (a few hundred or more) of sorted orientation angle combinations with respect to $E$ are fed back into the frequency calculation program so that actual natural frequencies are compared each other and made to ensure accuracy as the optimum solution.

### *3.3. A Procedure for the Plates with Many Layers*

In numerical experiments, it turned out on a standard PC that one faces the limit of all stacking combinations for the plate with many layers. For such cases, it is proposed to divide the search domain into sub-domains, which employs the idea of the layerwise optimization (LO) approach [7]. The LO method is based on the physical interpretation in the laminate theory that the bending stiffness is more governed by outer layers than inner ones, and introduces an assumption that the laminated plate can be optimized sequentially from the outer layers for the bending problem. This assumption is also available in deriving fiber orientation angles from lamination parameters since the parameters are defined by differences between cubic values of the normalized distance from the mid-plane to layer surfaces as equation (8) shown, and the outer layers have greater influence to a decision of parameter values. Thus the angles in the outer domains of the divided sub-domains would be determined first.

Figure 2 shows an example of the present optimization procedure for the symmetric 24-layered plate. Due to the plate symmetry, 12 layers are considered here.

*Step 0:* The half of plate is divided into three sub-domains, and each of them has four layers.

*Step 1:* Angles for outer four layers ($\theta_1$–$\theta_4$) in Sub-domain 1 are determined in this step using the method proposed in Section 3.2. All possible discrete lamination parameters are calculated only using outer four layers and other inner eight layers

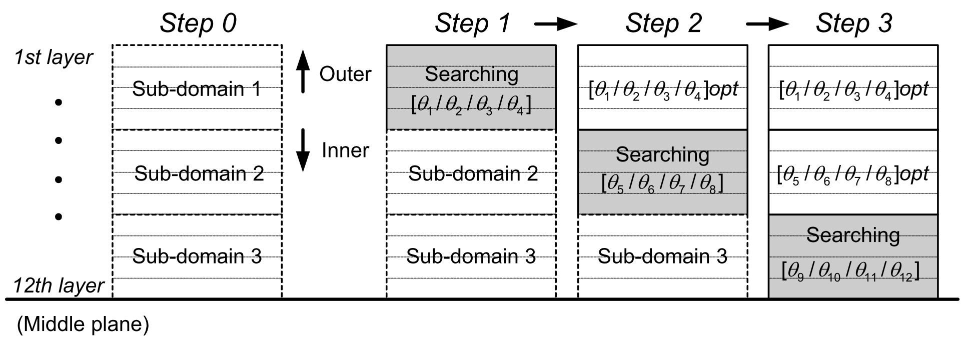


**Figure 2.** An angle deriving procedure from lamination parameters for symmetrically 24-layered plate.

are ignored. One of the possible parameters $W_1^{*(1)}$ in Step 1 are given by

$$W_1^{*(1)} = \sum_{k=1}^{4} (\xi_k^3 - \xi_{k-1}^3) \cos 2\theta_k, \tag{20}$$

where $\xi_k$ are the normalized distance from the plate mid-surface to the upper surface of $k$th layer given by (7) and the superscript in parenthesis denotes the step number. The $W_2^{*(1)}$, $W_3^{*(1)}$, and $W_4^{*(1)}$ are given similarly. Then the parameters $W_i^{*(1)}$ ($i = 1, 2, 3, 4$) are compared with the optimum lamination parameters $W_i$ by (19), and after the feedback process, the optimum angles for outer four layers $[\theta_1/\theta_2/\theta_3/\theta_4]_{\text{opt}}$ are obtained.

*Step 2:* In this step, angles in the middle four layers in Sub-domain 2 ($\theta_5$–$\theta_8$) are determined. All possible discrete lamination parameters $W_i^{*(2)}$ are calculated for all discrete possible angle combinations in the middle four layers, and inner four layers are still ignored. $W_1^{*(2)}$, in this step, is given by

$$W_1^{*(2)} = W_{1\,\text{opt}}^{*(1)} + \sum_{k=5}^{8} (\xi_k^3 - \xi_{k-1}^3) \cos 2\theta_k, \tag{21}$$

where $W_{1\,\text{opt}}^{*(1)}$ is the parameter corresponding to the optimum angles obtained in Step 1 $[\theta_1/\theta_2/\theta_3/\theta_4]_{\text{opt}}$, and $W_i^{*(2)}$ are calculated using outer eight layers. Then the comparison and feedback processes find out optimum angles $[\theta_5/\theta_6/\theta_7/\theta_8]_{\text{opt}}$ in the middle layers.

*Step 3:* The angles for the inner four layers $[\theta_9/\theta_{10}/\theta_{11}/\theta_{12}]_{\text{opt}}$ are determined using a similar way to Step 2, and finally the optimum whole lay-up $[\theta_1/\theta_2/\cdots/\theta_{12}]_{\text{s}}$ is extracted.

The present procedure is available for more than 24-layered plate by simply adding other steps to the above process and successfully reduces the whole domain optimization into the iteration of sub-domain optimization problems by using the LO idea.

## 4. Numerical Example Discussion

### 4.1. Maximizing the Fundamental Frequency of Square Plates

Frequency parameters are calculated from the frequency equation derived from (15). The elastic constants used in the following examples are taken for carbon/epoxy composite

$$E_L = 138 \text{ GPa}, \qquad E_T = 8.96 \text{ GPa}, \qquad G_{LT} = 7.1 \text{ GPa} \quad \text{and} \quad \nu_{LT} = 0.30.$$

In the convergence tests previously reported, the number of terms $M = N = 10$ for convergence are enough in (14) [26]. An increment of 5° is assumed for a discrete fiber orientation angle in each layer.

The first example deals with symmetrically 8-layered square ($a/b = 1$) plates. The boundary conditions of the plates are denoted by free (F), simply supported (S) and clamped (C) edge. The notation is applied counterclockwise to the plate edges from a left edge.

Table 1 presents sets of the lamination parameters optimized by one of mathematical programming techniques provided in the ADS program. The parameters are given for square plates with 21 different combinations of typical boundary con-

**Table 1.**
The optimum lamination parameters calculated by the gradient method for symmetrically 8-layered square plates ($a/b = 1$)

| B.C. | $W_1$ | $W_2$ | $W_3$ | $W_4$ |
|---|---|---|---|---|
| FFFF | −0.001 | −0.598 | 0.007 | 0.000 |
| SFFF | 0.023 | −1.000 | −0.032 | 0.026 |
| CFFF | 0.985 | 0.956 | −0.106 | −0.193 |
| SSFF | 0.008 | −1.002 | −0.306 | −0.003 |
| SCFF | −0.485 | −0.308 | 0.065 | −0.601 |
| CCFF | 0.011 | −0.957 | 0.388 | 0.211 |
| SFSF | 0.995 | 0.996 | 0.043 | 0.083 |
| SFCF | 0.999 | 0.996 | 0.014 | 0.028 |
| CFCF | 0.986 | 0.996 | 0.028 | 0.045 |
| SSSF | 0.987 | 0.981 | 0.033 | 0.041 |
| SCSF | 0.985 | 0.952 | −0.003 | 0.018 |
| SSCF | 0.980 | 0.951 | −0.142 | −0.263 |
| SCCF | 0.988 | 0.952 | −0.153 | −0.304 |
| CSCF | 0.999 | 0.998 | 0.016 | 0.033 |
| CCCF | 0.989 | 0.995 | 0.021 | 0.033 |
| SSSS | 0.020 | −1.001 | −0.014 | −0.002 |
| SSSC | −0.587 | −0.310 | −0.031 | 0.032 |
| SSCC | −0.001 | −1.001 | 0.022 | −0.023 |
| SCSC | −0.968 | 0.879 | −0.008 | 0.038 |
| CCCS | 0.988 | 0.953 | −0.003 | 0.002 |
| CCCC | 0.014 | 0.997 | 0.000 | 0.085 |

ditions. It is presumed that the parameter values in the table may contain slight numerical errors, because some parameter values close to zero (e.g., $W_1 = 0.001$) should be zero by physical inspection. This suggests the necessity of feedback to make the calculated parameters into the frequency calculation.

Table 2 presents the maximum frequency parameters for the first modes and corresponding stacking sequences obtained from the present approach for symmetrically 8-layered square plates. The boundary conditions are given for 21 different combinations of F, S and C, starting from a totally free plate FFFF to totally clamped plate CCCC. For FFFF and SFFF plates, rigid body modes of translation and rotation are ignored. The '$r$' in Table 2 denotes the ranking when stacking sequences are just sorted by using values of $E$ in (19). Among 21 combinations, specially orthotropic plates $[0/0/0/0]_s$ are found in nine cases, where a pair of two opposite edges are strongly constrained (e.g., <u>S</u>F<u>S</u>F, <u>S</u>F<u>C</u>F and <u>C</u>F<u>C</u>F) except for a cantilever type plate of CFFF plate. A similar remark may be made on $[90/90/90/90]_s$ for S<u>C</u>S<u>C</u> plate. The other cases exhibit some interactive influences from constraints of adjacent edges.

The present results are compared to those obtained by the LO method [7]. The comparison reveals that the present method gives slightly higher frequencies for

**Table 2.**
The maximum frequency parameters and optimum stacking sequences by the present approach and the LO method for symmetrically 8-layered square plates ($a/b = 1$)

| B.C. | $\Omega_{Present}$ | Optimum sequence | $r$ | $\Omega_{LO}$ | Optimum sequence |
|---|---|---|---|---|---|
| FFFF | <u>38.98</u> | $[40/-35/-85/-80]_s$ | 42 | 37.84 | $[55/-50/10/-80]_s$ |
| SFFF | <u>21.70</u> | $[-45/45/40/40]_s$ | 3 | 21.69 | $[45/-40/-45/-45]_s$ |
| CFFF | 13.79 | $[0/0/0/0]_s$ | 3525 | 13.79 | $[0/0/0/0]_s$ |
| SSFF | 11.29 | $[-45/45/-45/45]_s$ | 1 | 11.29 | $[-45/45/-45/45]_s$ |
| SCFF | 16.40 | $[70/-45/65/70]_s$ | 2123 | 16.40 | $[70/-45/70/65]_s$ |
| CCFF | 19.03 | $[45/-45/45/-55]_s$ | 1792 | 19.03 | $[45/-45/45/-35]_s$ |
| SFSF | 38.69 | $[0/0/0/0]_s$ | 364 | 38.69 | $[0/0/0/0]_s$ |
| SFCF | 60.47 | $[0/0/0/0]_s$ | 18 | 60.47 | $[0/0/0/0]_s$ |
| CFCF | 87.77 | $[0/0/0/0]_s$ | 113 | 87.77 | $[0/0/0/0]_s$ |
| SSSF | 39.84 | $[0/0/0/0]_s$ | 165 | 39.84 | $[0/0/0/0]_s$ |
| SCSF | 40.28 | $[0/0/0/0]_s$ | 186 | 40.28 | $[0/0/0/0]_s$ |
| SSCF | 61.49 | $[-5/0/-5/-5]_s$ | 35 | 61.49 | $[-5/0/-5/-5]_s$ |
| SCCF | 61.88 | $[-5/-5/0/0]_s$ | 2 | 61.88 | $[-5/-5/0/0]_s$ |
| CSCF | 88.41 | $[0/0/0/0]_s$ | 28 | 88.41 | $[0/0/0/0]_s$ |
| CCCF | 88.63 | $[0/0/0/0]_s$ | 58 | 88.63 | $[0/0/0/0]_s$ |
| SSSS | 56.32 | $[-45/45/45/45]_s$ | 1 | 56.32 | $[45/-45/-45/-45]_s$ |
| SSSC | 66.73 | $[-65/60/60/60]_s$ | 29 | 66.73 | $[-65/60/60/60]_s$ |
| SSCC | <u>72.19</u> | $[45/-45/-45/-45]_s$ | 3 | 72.06 | $[-45/45/45/45]_s$ |
| SCSC | 90.89 | $[90/90/90/90]_s$ | 2139 | 90.89 | $[90/90/90/90]_s$ |
| CCCS | 91.99 | $[0/0/0/0]_s$ | 137 | 91.99 | $[0/0/0/0]_s$ |
| CCCC | 93.67 | $[0/90/90/90]_s$ | 50 | 93.67 | $[0/90/90/90]_s$ |

**Table 3.**
The maximum frequency parameters and optimum stacking sequences for symmetrically 16-, 24- and 32-layered square plates ($a/b = 1$)

| B.C. | Ω | Optimum sequence |
|---|---|---|
| 16-layer | | |
| SSSS | 56.53 | $[-45/45/45/-45/45/-45/-45/45]_s$ |
| CCCC | 93.67 | $[90/0/0/90/0/90/90/0]_s$ |
| SCFF | 16.31 | $[75/-45/60/75/75/75/75/60]_s$ |
| SSSC | 66.89 | $[-60/60/60/-60/60/-60/-60/60]_s$ |
| 24-layer | | |
| SSSS | 56.53 | $[-45/45/45/-45/45/-45/-45/45/45/-45/-45/-45]_s$ |
| CCCC | 93.67 | $[0/90/90/0/90/0/0/90/90/0/90/90]_s$ |
| SCFF | 16.31 | $[75/-45/75/60/75/75/75/-45/75/-45/75/-60]_s$ |
| SSSC | 66.89 | $[60/-60/-60/60/-60/60/60/-60/60/-60/-60/-60]_s$ |
| 32-layer | | |
| SSSS | 56.53 | $[-45/45/45/-45/45/-45/-45/45/45/-45/-45/45/-45/45/45/-45]_s$ |
| CCCC | 93.67 | $[0/90/90/0/90/0/0/90/90/0/90/0/0/90/90/0]_s$ |
| SCFF | 16.32 | $[75/-45/75/60/75/75/-45/75/75/75/75/-45/75/-45/60/60]_s$ |
| SSSC | 66.89 | $[60/-60/-60/60/-60/60/60/60/-60/60/60/-60/60/-60/-60/60]_s$ |

three boundary conditions (FFFF, SFFF and SSCC: underlined) than those of the LO method. These three boundary conditions require feedback less than 50 trials. The CFFF plate has the lowest rank of 3525, but this rank is still within less than 0.2% of all the combinations of about 1.68 million. It means that the errors caused in the lamination parameter optimization by using an ADS program are within a negligible range if one needs a nearly optimum solution with an accuracy of about 0.2% of the maximum. Although very slight discrepancies are found for FFFF, SFFF and SSCC plates, sets of optimum solutions obtained by both methods are generally agree well in the table.

Table 3 shows the optimum solutions of symmetric 16, 24 and 32-layer square plates with an increment 15°. This optimization is made to verify that the present method still works well for laminated plates with more layers than eight by using the technique explained in Section 3.3. The boundary conditions are chosen for SSSS, CCCC, SCFF and SSSC plates, and the optimum fiber orientation angles are similar to those of 8-layered plates.

### *4.2. Maximizing the Fundamental Frequency of Rectangular Plates*

Tables 4 and 5 present the optimum solutions for different aspect ratios ($a/b = 1.5$ and 2, respectively) in the same format as Table 2. The present results in Table 4 give slightly higher frequencies than the LO results at four boundary conditions (FFFF, SFFF, SSFF and SSSS: underlined), but lower ones at two boundary conditions (SSSF and SCCF). The boundary conditions with higher frequencies in Table 5 are two (FFFF and SSCF), but lower ones are four (SFFF, CFFF, CCFF and SCSF). It

**Table 4.**
The maximum frequency parameters and optimum stacking sequences for symmetrically 8-layered plates by the present approach and the LO method ($a/b = 1.5$)

| B.C. | $\Omega_{Present}$ | Optimum sequence | $r$ | $\Omega_{LO}$ | Optimum sequence |
|---|---|---|---|---|---|
| FFFF | <u>54.71</u> | $[40/-15/-55/-25]_s$ | 3221 | 53.19 | $[-25/30/-60/35]_s$ |
| SFFF | <u>31.71</u> | $[45/-40/-35/15]_s$ | 270 | 31.63 | $[-45/35/45/-30]_s$ |
| CFFF | 13.79 | $[0/0/0/0]_s$ | 1166 | 13.79 | $[0/0/0/0]_s$ |
| SSFF | <u>16.79</u> | $[-45/45/45/-45]_s$ | 1876 | 16.78 | $[-40/45/-45/45]_s$ |
| SCFF | 32.93 | $[85/80/85/85]_s$ | 428 | 32.93 | $[85/80/85/85]_s$ |
| CCFF | 33.88 | $[80/85/80/80]_s$ | 2258 | 33.88 | $[80/85/80/80]_s$ |
| SFSF | 38.67 | $[0/0/0/0]_s$ | 21 | 38.67 | $[0/0/0/0]_s$ |
| SFCF | 60.45 | $[0/0/0/0]_s$ | 5 | 60.45 | $[0/0/0/0]_s$ |
| CFCF | 87.76 | $[0/0/0/0]_s$ | 98 | 87.76 | $[0/0/0/0]_s$ |
| SSSF | 41.42 | $[-20/25/20/20]_s$ | 521 | <u>41.50</u> | $[15/-20/-20/-20]_s$ |
| SCSF | 45.69 | $[45/-40/-40/-45]_s$ | 102 | 45.69 | $[-45/40/40/45]_s$ |
| SSCF | 62.90 | $[-5/-5/-5/-5]_s$ | 1098 | 62.90 | $[-5/-5/-5/-5]_s$ |
| SCCF | 64.23 | $[-5/-5/-10/-5]_s$ | 2925 | <u>64.24</u> | $[-5/-5/-5/-5]_s$ |
| CSCF | 89.19 | $[0/0/0/0]_s$ | 26 | 89.19 | $[0/0/0/0]_s$ |
| CCCF | 89.98 | $[0/0/0/0]_s$ | 19 | 89.98 | $[0/0/0/0]_s$ |
| SSSS | <u>93.84</u> | $[-65/60/65/65]_s$ | 216 | 93.83 | $[70/-65/-65/-65]_s$ |
| SSSC | 140.0 | $[90/90/90/90]_s$ | 372 | 140.0 | $[90/90/90/90]_s$ |
| SSCC | 140.9 | $[90/90/90/90]_s$ | 290 | 140.9 | $[90/90/90/90]_s$ |
| SCSC | 200.4 | $[90/90/90/90]_s$ | 248 | 200.4 | $[90/90/90/90]_s$ |
| CCCS | 142.1 | $[90/90/90/90]_s$ | 287 | 142.1 | $[90/90/90/90]_s$ |
| CCCC | 201.9 | $[90/90/90/90]_s$ | 91 | 201.9 | $[90/90/90/90]_s$ |

should be noted that CFFF (cantilever) in Table 5 ranks the lowest in all conditions raging Tables 2, 4 and 5. The number of feedback is 3809 and still around 0.2%, but from physical view point, it is expected that optimum stacking sequence of CFFF is $[0/0/0/0]_s$ and the result from the LO is so.

From an overview of Tables 2, 4 and 5, the optimum stacking sequences are $[0/0/0/0]_s$ at four boundary conditions (SFSF, CFCF, CSCF and CCCF) in all tables. This means that the boundary condition influences the optimum stacking sequence more than the aspect ratio of plate in these boundary conditions. On the other hand, for six boundary conditions without free edges (SSSS, SSSC, SSCC, SCSC, CCCS and CCCC), only SCSC in Table 2 shows $[90/90/90/90]_s$ for the optimum stacking sequence, but five boundary conditions (except for SSSS) in Table 4 and all of them in Table 5 give $[90/90/90/90]_s$. Thus, the aspect ratio plays a more influential role than the boundary conditions to optimum stacking sequence in these six boundary conditions.

### *4.3. Maximizing the Difference between the First Two Frequencies*

It is generally important to make the first (fundamental) natural frequencies higher to avoid the resonance of laminated plates. However, when the first frequency of

**Table 5.**
The maximum frequency parameters and optimum stacking sequences for symmetrically 8-layered plates by the present approach and the LO method ($a/b = 2$)

| B.C. | $\Omega_{\text{Present}}$ | Optimum sequence | $r$ | $\Omega_{\text{LO}}$ | Optimum sequence |
|---|---|---|---|---|---|
| FFFF | <u>65.60</u> | $[10/-35/55/70]_s$ | 3818 | 65.25 | $[5/-40/50/50]_s$ |
| SFFF | 38.59 | $[-35/25/35/30]_s$ | 3752 | <u>38.80</u> | $[-35/25/35/30]_s$ |
| CFFF | 12.25 | $[15/-25/-20/35]_s$ | 3809 | <u>13.79</u> | $[0/0/0/0]_s$ |
| SSFF | 22.20 | $[-40/45/45/-45]_s$ | 3678 | 22.20 | $[-40/45/45/-45]_s$ |
| SCFF | 57.06 | $[85/85/85/85]_s$ | 37 | 57.06 | $[85/85/85/85]_s$ |
| CCFF | 57.67 | $[85/80/85/85]_s$ | 3781 | <u>57.71</u> | $[85/85/85/85]_s$ |
| SFSF | 38.66 | $[0/0/0/0]_s$ | 18 | 38.66 | $[0/0/0/0]_s$ |
| SFCF | 60.44 | $[0/0/0/0]_s$ | 4 | 60.44 | $[0/0/0/0]_s$ |
| CFCF | 87.74 | $[0/0/0/0]_s$ | 76 | 87.74 | $[0/0/0/0]_s$ |
| SSSF | 48.52 | $[-35/35/35/40]_s$ | 27 | 48.52 | $[35/-35/-35/-40]_s$ |
| SCSF | 63.94 | $[55/-55/-55/-55]_s$ | 1652 | <u>64.09</u> | $[-60/55/55/55]_s$ |
| SSCF | <u>64.90</u> | $[-10/-10/25/-10]_s$ | 572 | 64.84 | $[-10/0/-5/25]_s$ |
| SCCF | 71.37 | $[-30/55/60/-35]_s$ | 496 | 71.37 | $[-30/55/60/-35]_s$ |
| CSCF | 90.28 | $[0/0/0/0]_s$ | 411 | 90.28 | $[0/0/0/0]_s$ |
| CCCF | 92.28 | $[0/0/0/0]_s$ | 320 | 92.28 | $[0/0/0/0]_s$ |
| SSSS | 159.9 | $[90/90/90/90]_s$ | 177 | 159.9 | $[90/90/90/90]_s$ |
| SSSC | 245.7 | $[90/90/90/90]_s$ | 372 | 245.7 | $[90/90/90/90]_s$ |
| SSCC | 246.4 | $[90/90/90/90]_s$ | 810 | 246.4 | $[90/90/90/90]_s$ |
| SCSC | 353.9 | $[90/90/90/90]_s$ | 21 | 353.9 | $[90/90/90/90]_s$ |
| CCCS | 247.1 | $[90/90/90/90]_s$ | 499 | 247.1 | $[90/90/90/90]_s$ |
| CCCC | 354.9 | $[90/90/90/90]_s$ | 159 | 354.9 | $[90/90/90/90]_s$ |

a laminated plate is relatively low against the external excitation frequency, one may need to maximize the difference between the first and second frequencies in order to avoid structural resonance. For this purpose, the maximum difference is calculated by the same method in the present frequency optimization. When the first and second frequencies are denoted by $\Omega_1$ and $\Omega_2$, respectively, the optimum lamination parameters for the difference are calculated by the ADS program just by changing the objective function to the difference $(\Omega_2 - \Omega_1)$.

The procedure is almost the same to determine the optimum stacking sequence. The result of optimization is shown in Table 6. The plates are 8-layered square ($a/b = 1$) plates, and the boundary conditions used in the example are CFFF, SSFF, SSSS, SCFF, CFCF and CCCC. For each condition, the ranking is high and, thus, the optimum parameters are calculated accurately to maximize the differences.

### *4.4. Minimizing the Difference between the Target and Actual Frequencies*

The method is applied to a design problem for finding the optimum stacking sequence to give any frequencies in the range between the maximum and minimum frequencies. The result of optimization is given in Table 7 for the square plates

**Table 6.**
The maximum differences between the first and second frequencies for symmetrically 8-layered square plates ($a/b = 1$)

| B.C. | $\Omega_2 - \Omega_1$ | $\Omega_1$ | $\Omega_2$ | Optimum sequence | $r$ |
|---|---|---|---|---|---|
| CFFF | 18.45 | 6.458 | 24.91 | $[-50/50/50/50]_s$ | 24 |
| SSFF | 38.97 | 5.337 | 44.31 | $[-5/85/85/85]_s$ | 837 |
| SSSS | 71.44 | 44.31 | 115.8 | $[0/90/90/90]_s$ | 32 |
| SCFF | 43.18 | 10.14 | 53.32 | $[0/85/5/85]_s$ | 2880 |
| CFCF | 21.53 | 36.83 | 58.36 | $[55/-55/-55/-55]_s$ | 11 |
| CCCC | 89.86 | 93.67 | 183.5 | $[0/90/90/90]_s$ | 137 |

**Table 7.**
The target frequencies and optimum stacking sequences for symmetrically 8-layered square plates ($a/b = 1$)

| B.C. | $\Omega_{target}$ | $\Omega_1$ | Optimum sequence | $r$ |
|---|---|---|---|---|
| SSSS | 50.00 | 50.00 | $[15/-40/-80/80]_s$ | 77 |
| CFFF | 13.00 | 13.00 | $[10/-20/-10/-45]_s$ | 374 |
| CCCC | 90.00 | 90.01 | $[10/30/80/15]_s$ | 127 |

($a/b = 1$) with symmetric 8-layers. The target frequencies of SSSS, CFFF and CCCC are set to 50.00, 13.00 and 90.00, respectively.

The obtained frequencies calculated by the present method are quite close to the target frequencies and the ranking is also high. Thus, it becomes clear that the present optimization method is quite accurate and useful to get the target frequencies of laminated plates.

Figure 3 shows the first vibration modes for the optimum stacking sequences with maximum natural frequencies and target frequencies. Although the SSSS plate gives the same mode shapes, CFFF and CCCC plates form different mode shapes between the plate with maximum frequencies and target frequencies. The latter plates have skewed shapes when they are compared with the former ones. These skewed shapes may be influenced by the optimum stacking sequences composed of various angles.

## 5. Conclusions

In this work, a discrete optimization method was proposed to obtain the fiber orientation angles from the optimum lamination parameters calculated by mathematical programming. The present method consists in the approach that the fiber orientation angles are determined by minimizing the difference between the optimum

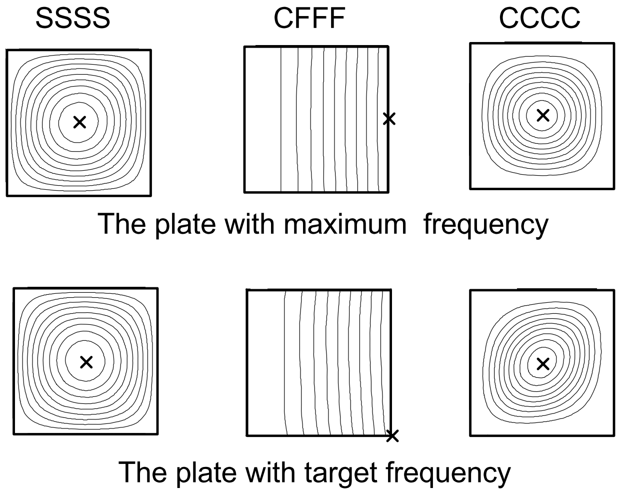


**Figure 3.** The first vibration mode shapes of symmetrically 8-layered square plates with optimum stacking sequences for the maximum frequency and target frequency.



lamination parameters and parameters obtained for all combinations of lamination parameters.

The idea of the present method is quite simple and straightforward. It is applicable to various procedures that use lamination parameters for determining the optimum stacking sequences. A feedback concept is explained to get accurate optimum solutions and the quick sorting technique results in reduction of computation time. For cases with many layers, a technique which employs the LO idea and divides a many layer problem into sub-domain problems is given to avoid the computation problem.

In numerical examples, accuracy is established by comparing the present results with other results and the method is extended to obtain results for maximizing the difference of the first and second frequencies and for tuning the frequency to the target value in the range between the maximum and minimum frequencies.

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